# Machine Learning

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Lecture Notes

Introduction

# What is Machine Learning?

Simon Says:

"Learning denotes changes in the system that enable the system to do the same task ... more effectively next time."

Example Tasks:

- Classify an object as an instance (or non-instance) of a general concept. (Inductive Concept Learning.)
- Solve a search problem. (Speedup Learning.)

- Learning by being told:

  *Teacher states the rules of grammar.*

  *Knowledge Compilation.*

- Learning from examples:

  *Teacher shows examples of good and bad sentences.*

  *Inductive Learning.*

- Learning by discovery:

  *Student talks and teacher corrects his grammar.*

  *Automated Theory Formation.*

# Inductive Argument

## Given:

$$
\begin{array}{lll}
P(a_1) \wedge Q(a_1) & \neg P(b_1) \wedge \neg Q(b_1) & \neg P(c_1) \wedge Q(c_1) \\
P(a_2) \wedge Q(a_2) & \neg P(b_2) \wedge \neg Q(b_2) & \neg P(c_2) \wedge Q(c_2) \\
\ldots & & \\
P(a_l) \wedge Q(a_l) & \neg P(b_m) \wedge \neg Q(b_m) & \neg P(c_n) \wedge Q(c_n)
\end{array}
$$

## Conclude:

$$(\forall x) P(x) \Rightarrow Q(x)$$

# The Old Problem of Induction

- Why are inductive arguments justified?
- Hume: Because they have worked in the past.
- Russell: That's using induction to justify induction.
- Goodman: Inductive arguments are not justified.

# The New Problem of Induction

How does an intelligent agent choose among many possible inductive generalizations of his observations?

Examples:

$$Emerald(a) \wedge Green(a) \text{ (Sunday).}$$
$$Emerald(b) \wedge Green(b) \text{ (Monday).}$$
$$Emerald(c) \wedge Green(c) \text{ (Tuesday).}$$

Generalizations:

$$(\forall x)Emerald(x) \Rightarrow Green(x)$$
$$(\forall x)Emerald(x) \Rightarrow Grue(x)$$

An object is "grue" if it is green on Sunday, Monday and Tuesday, but is blue for the rest of the week.

# Inductive Bias

"Any criteria for choosing one concept description over another, other than strict consistency with the training examples." (Mitchell)

- E.g., A biased concept description language.
- E.g., A biased learning algorithm.

# Occam's Razor

- Choose the simplest hypothesis.
- That accounts for the observation.
- "Green" is simpler than "Grue"
- So choose a hypothesis involving the term "Green".

# Concept Learning Problem Definition

- Given Training Data:
  - Positive Examples:
    $(ObjectDescription, +Label)$
  - Negative Examples:
    $(ObjectDescription, -Label)$
- Find rule for predicting whether future examples are positive or negative.
- "Concept Membership Rule".

# Definitions

- Instance Description Language:
  - Language for describing example objects.
  - E.g., Boolean, Integer or Real Vector.
- Concept Description Language:
  - Language for describing concepts.
  - E.g., Conjunctive: $\wedge(v_i = k_i)$.

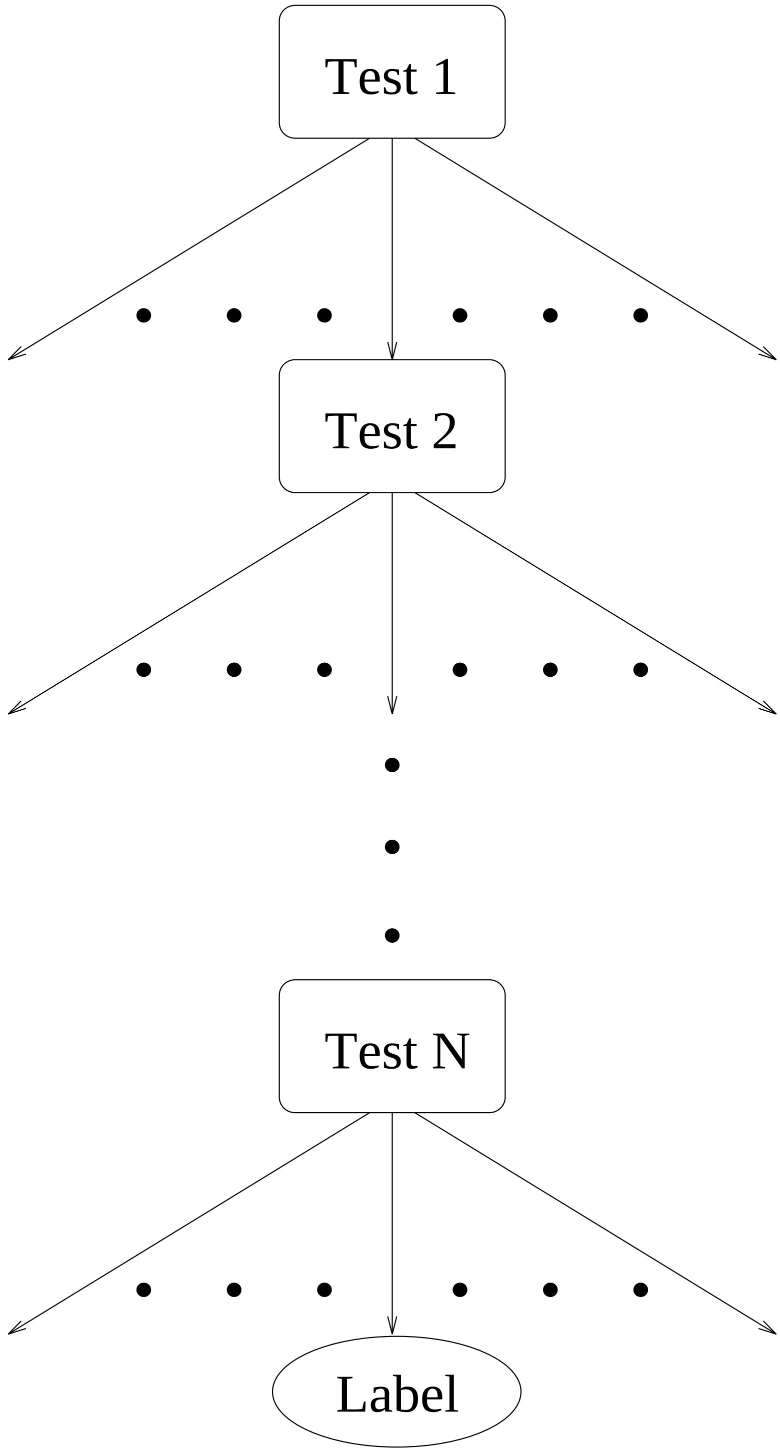
# Decision Trees

- A concept description language.
- For instances represented as feature vectors.
- Each internal node checks the value of a feature.
- Branches are labeled by possible values.
- Leaves are labeled:

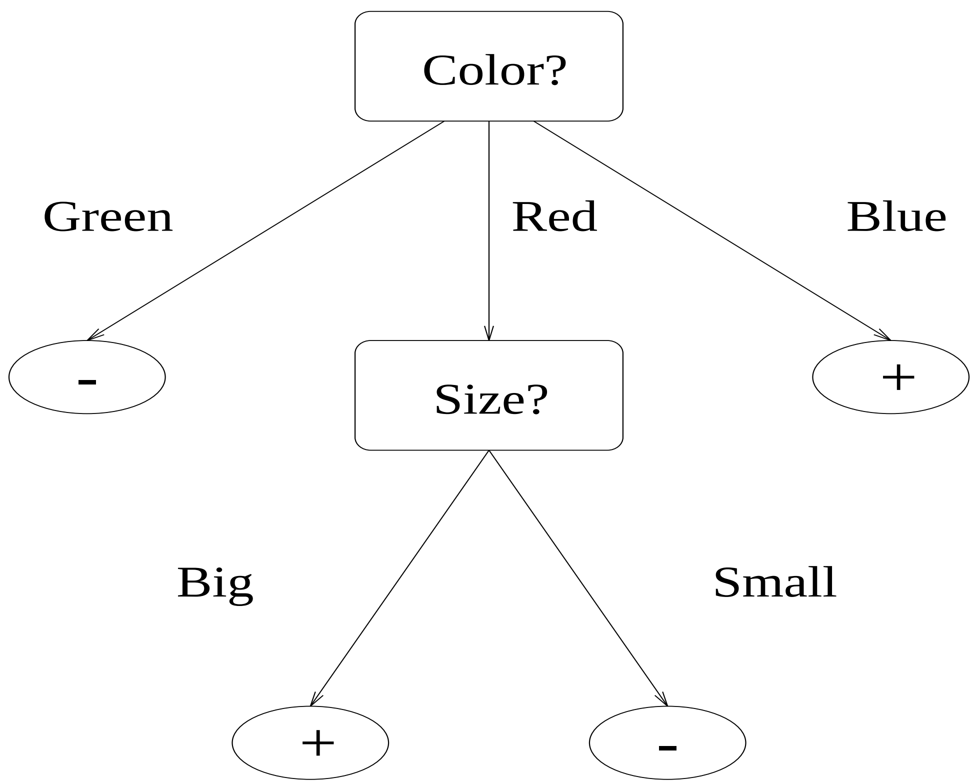
  "+" indicates member of the concept.

  "-" indicates not a member of the concept.

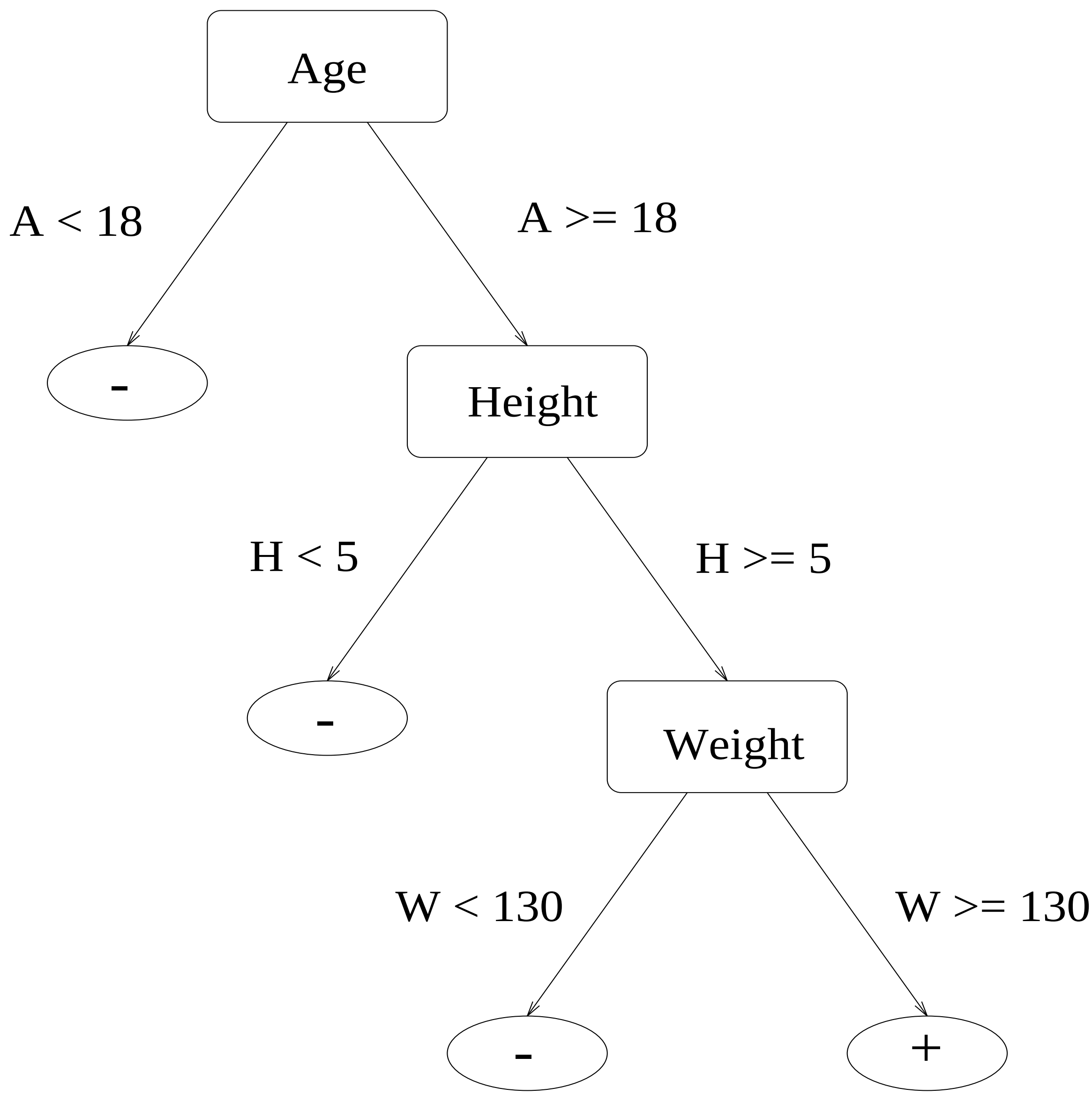
# Generic Decision Tree



# A Decision Tree for Discrete Feature Vectors



# A Decision Tree for Continuous Feature Vectors



- Goal: Find a smallest tree that correctly classifies all the training examples.
- NP Hard: (Hyafil and Rivest, 1976).
- We must use heuristics if CPU time is limited.

# ID3: Greedy Algorithm to Find Small Decision Trees

Given a set of labeled instances:

1. Find a feature that "best" divides the instances into uniform sets.
2. Recursively call ID3 on each subset.

*What does "best" mean?*

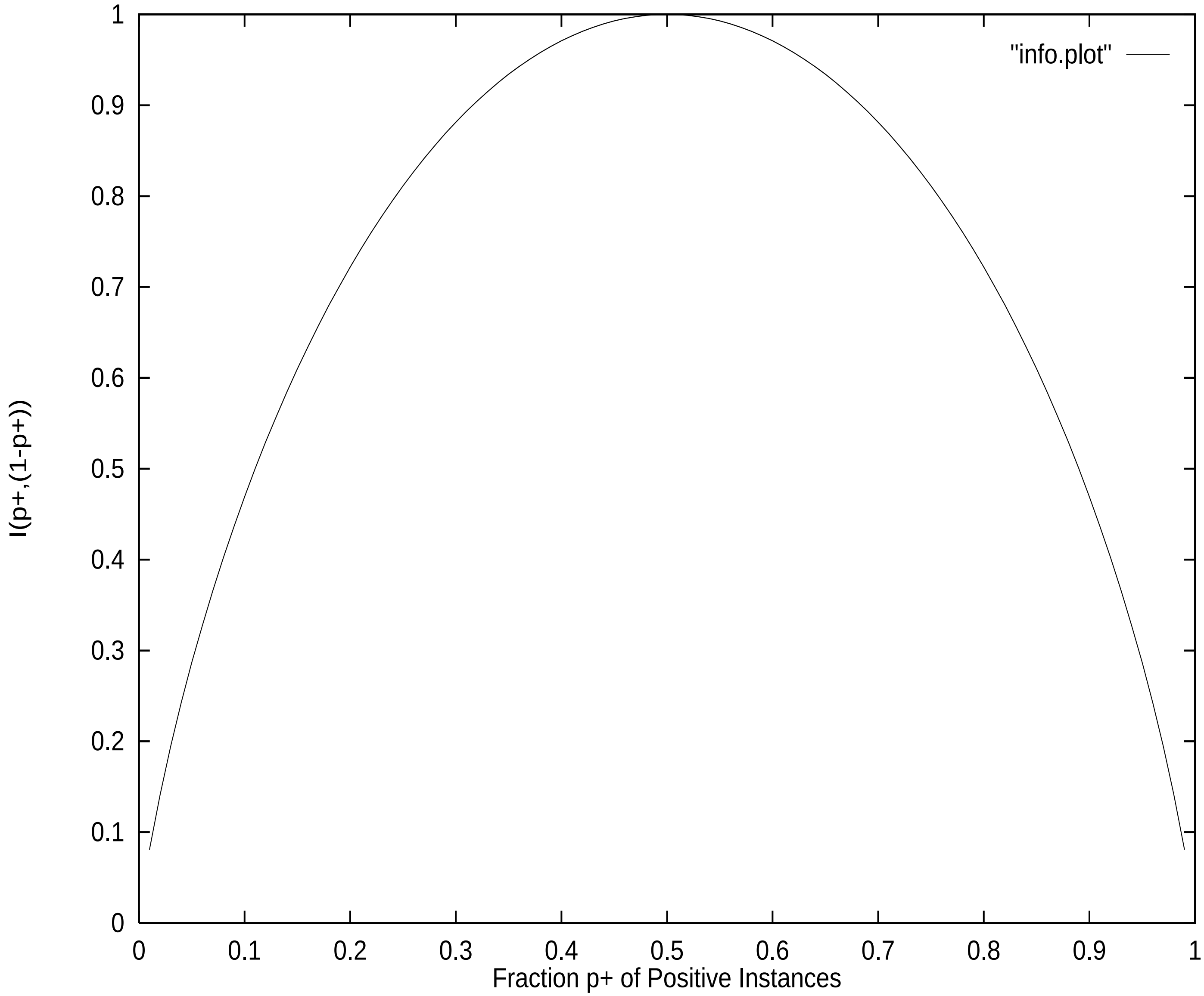
- Let $S$ be a set of unclassified instances.
- Assume we know the fractions of positive and negative instances in $S$:

  $p^{+}$ = Fraction of positive instances.

  $p^{-}$ = Fraction of negative instances.
- Now someone tells us the classification of some instance in set $S$.
- What is the *information value* of this new fact?

  $$I(p^{+}, p^{-}) = -(p^{+}) log_2(p^{+}) - (p^{-}) log_2(p^{-})$$
- The information value of the fact is the *Entropy* of the instances.

# Information Content Function



# Finding a "Best" Feature

1. For each feature $f$ do:
   (a) Use $f$ to partition instances into sets $S_1, \ldots, S_n$.
   (b) For each set $S_i$, determine $p_i^+$ and $p_i^-$.
   (c) Let
   $$Gain(f) = I(p^+, p^-) - \Sigma_{i=1}^{n} \left(\frac{|S_i|}{|S|}\right) I(p_i^+, p_i^-).$$
2. Choose a feature $f$ with highest value of $Gain(f)$.

# ID3: Quinlan (Discrete Feature Vectors)

```
ID3(INSTANCES, FEATURES):

1. If all INSTANCES are positive, then Return(Positive-Leaf).
2. If all INSTANCES are negative, then Return(Negative-Leaf).
3. Let BEST = Maxarg (f in FEATURES) Gain(f).
4. Let R be the root of a decision tree splitting on BEST.
5. For each value V of BEST do:
   a. Let S = Subset of INSTANCES with BEST = V.
   b. Attach to R the subtree ID3(S,FEATURES - {BEST}).
```

# An ID3 Example

| | Color | Shape | Size | Class |
|---|---|---|---|---|
| 1 | Red | Square | Big | + |
| 2 | Blue | Square | Big | + |
| 3 | Red | Round | Small | - |
| 4 | Green | Square | Small | - |
| 5 | Red | Round | Big | + |
| 6 | Green | Round | Big | - |

Initial Call:

```
ID3({1,2,3,4,5,6},{Color, Shape,Size})
```

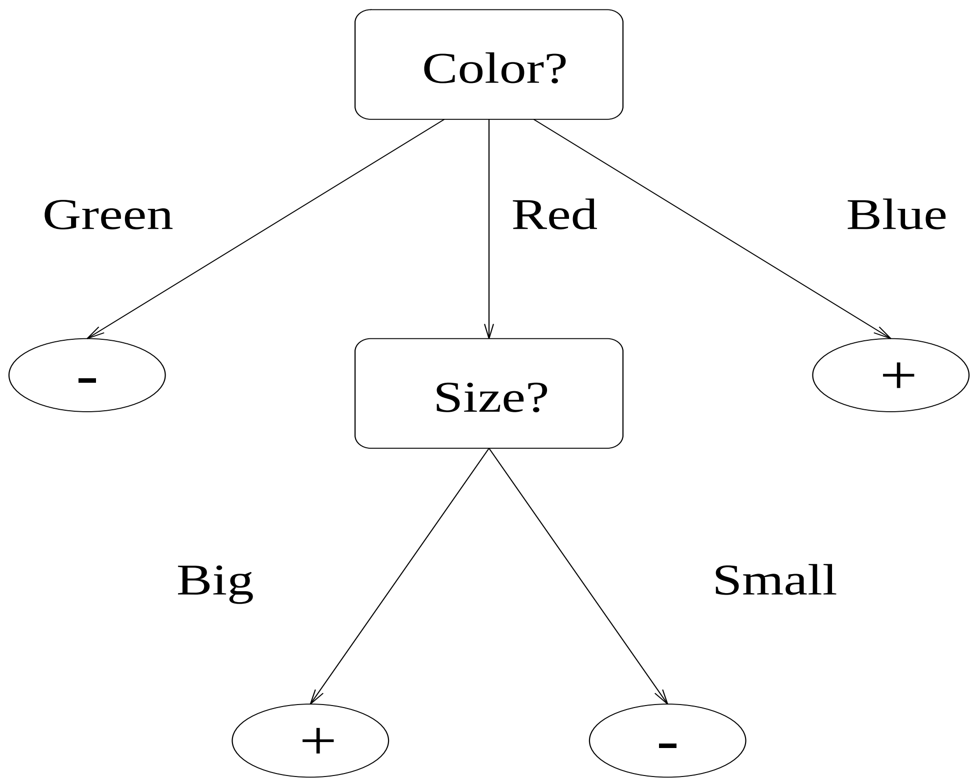
$$\begin{aligned} Gain(Color) &= I(3/6, 3/6) \\ &\quad -(3/6) \cdot I(2/3, 1/3) \\ &\quad -(1/6) \cdot I(1/1, 0/1) \\ &\quad -(2/6) \cdot I(0/2, 2/2) \\ &= 1 - 0.459 = \mathbf{0.541} \end{aligned}$$

$$\begin{aligned} Gain(Shape) &= I(3/6, 3/6) \\ &\quad -(3/6) \cdot I(2/3, 1/3) \\ &\quad -(3/6) \cdot I(1/3, 2/3) \\ &= 1 - 0.918 = \mathbf{0.082} \end{aligned}$$

$$\begin{aligned} Gain(Size) &= I(3/6, 3/6) \\ &\quad -(2/3) \cdot I(3/4, 1/4) \\ &\quad -(1/3) \cdot I(0/2, 2/2) \\ &= 1 - 0.541 = \mathbf{0.459} \end{aligned}$$

Color is best!

# Tree Returned by ID3



# Generating Rules from Decision Trees

- One rule for each path from root to a leaf node.
- Antecedent: Conjunction of all decisions on path.
- Consequent: Label of the leaf node.

$$Color = Green \Rightarrow -$$

$$Color = Blue \Rightarrow +$$

$$Color = Red \wedge Size = Big \Rightarrow +$$

$$Color = Red \wedge Size = Small \Rightarrow -$$

# C4.5: Quinlan (Continuous Feature Vectors)

- Similar to handling of discrete feature vectors.
- For each internal, splitting node:
  - Choose best feature.
  - Choose direction $<$ or $\geq$ of test.
  - Choose threshold $k$ of test: $f < k$ or $f \geq k$.

# Estimating Accuracy of a Concept Description

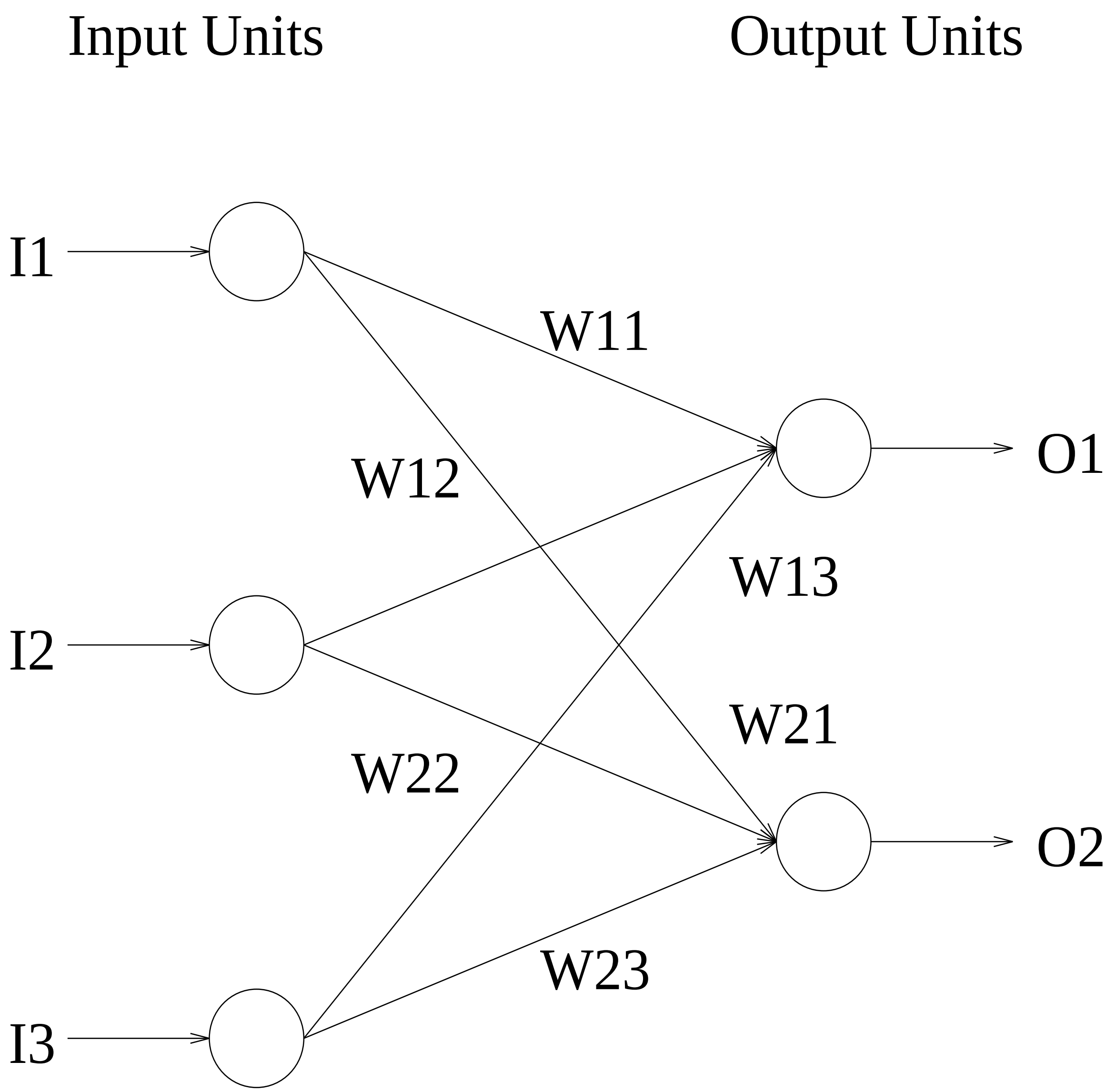
- Data Rich: Separate Training and Test Sets.
  - Select a random subset of the training examples.
  - Withold it from the learning algorithm.
  - Use it as an unbiased test set.
- Data Poor: Cross Validation
  - Divide data into $n$ subsets.
  - Learn one concept description for each collection of $n - 1$ subsets.
  - Test each concept description on the corresponding witheld subset.
  - Use average of $n$ error rates as an estimate of the accuracy when learning from all the data.

- Also Known As:
  - Connectionist Machines.
  - Parallel Distributed Processing.
- Early Work: Perceptrons.
- Current Work: Backpropagation.

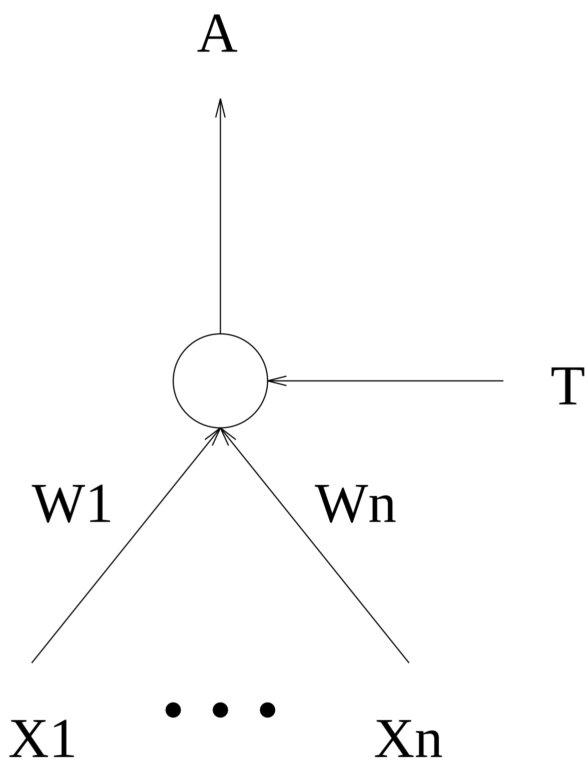
# Perceptron

- Rosenblatt, 1958.
- Very simple computing device.
- Very simple learning device.
- Inspired by rough analogy with neuron.

# Perceptron

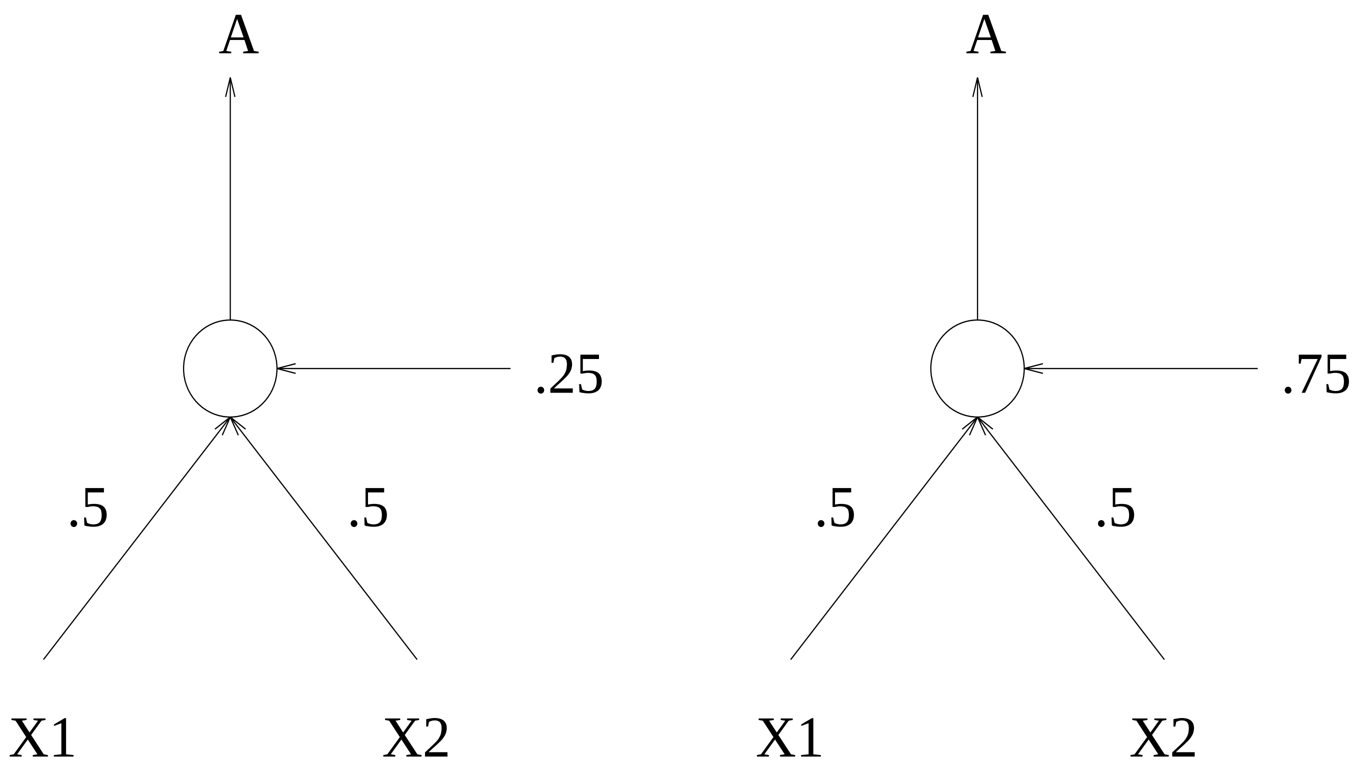


# Perceptron Output Function




$$A = \begin{cases} 1 & \text{If } W_1X_1 + \ldots + W_nX_n > T \\ 0 & \text{Otherwise.} \end{cases}$$

# Examples



Notice that

$$W_1X_1 + \ldots + W_nX_n > T$$

is equivalent to

$$W_1X_1 + \ldots + W_nX_n - T > 0$$

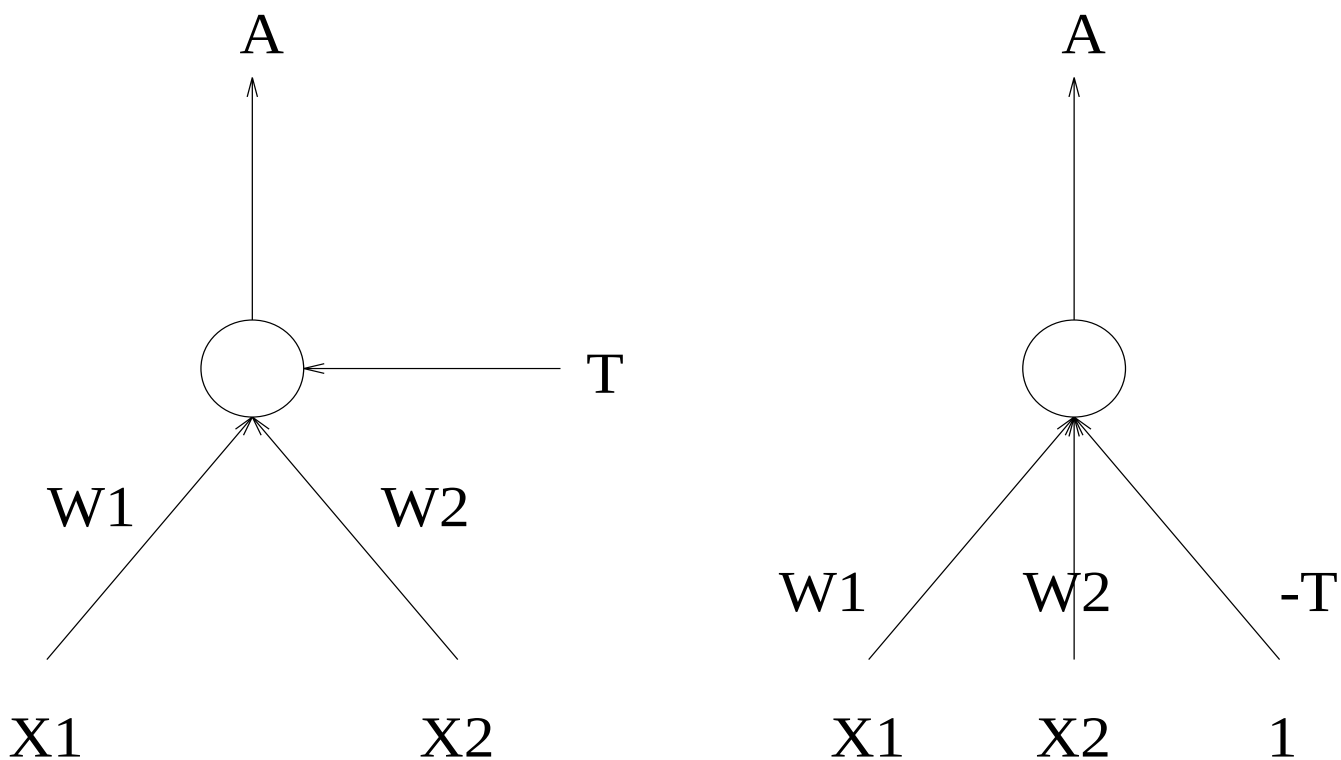
or

$$W_1X_1 + \ldots + W_nX_n - T \cdot 1 > 0$$

# Thresholds as Weights

- Pretend each node has an extra input whose value is always 1 and whose weight is $-T$, called the "bias".
- Learning updates the bias just like all the other weights.

# Thresholds as Weights



# Learning in Neural Networks

- Learn values of weights from I/O pairs.
- Start with random weights.
- Load training example's input.
- Observe computed output.
- Compare to desired output.
- Modify weights to reduce difference.
- Iterate over all training examples.
- Terminate when weights stop changing.

# Perceptron Learning Rule

$$\Delta W_i = \eta(D - A)X_i$$

$X_i$ is a node's input.

$W_i$ is the corresponding weight.

$\Delta W_i$ is the change in weight.

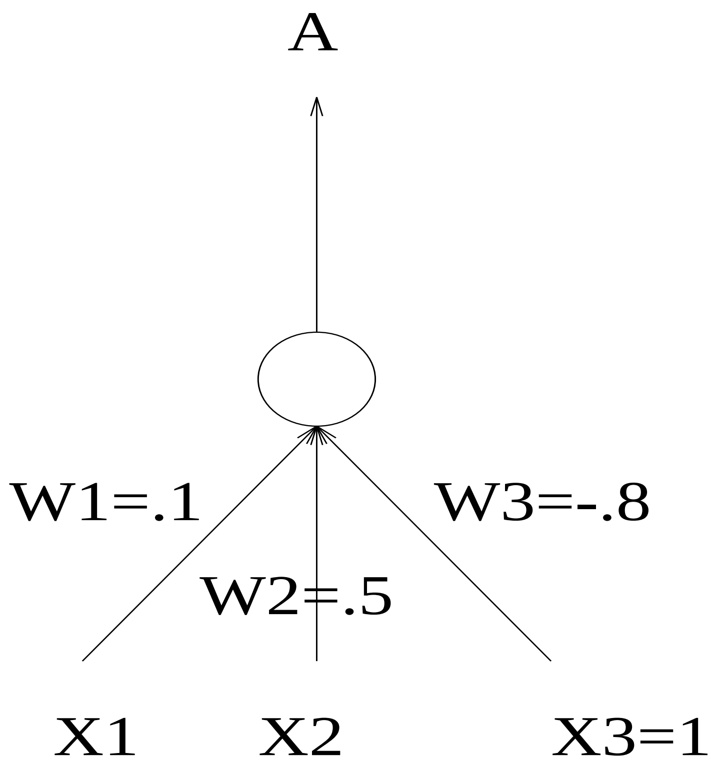
$D$ is the desired output.

$A$ is the actual observed output.

$\eta$ is the learning rate.

# Learning the *Or* Function

$$\Delta W_i = .2(D - A)X_i$$



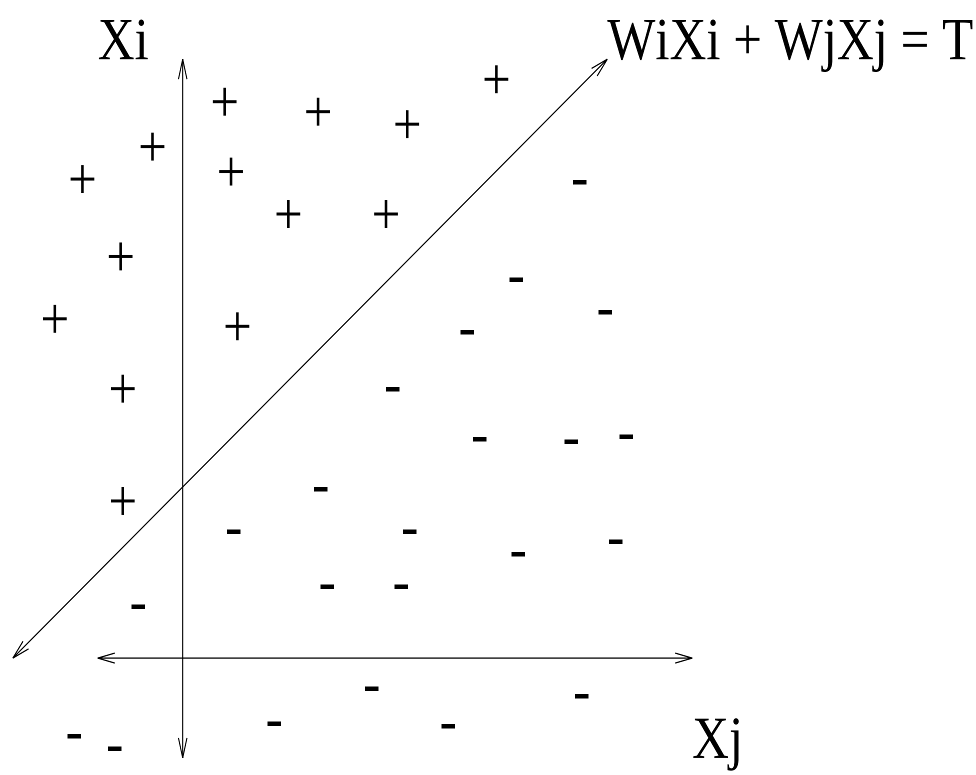
# Learning the *Or* Function

| $X_1$ | $X_2$ | $D$ | $A$ | $\Delta W_1$ | $W_1$ | $\Delta W_2$ | $W_2$ | $\Delta W_3$ | $W_3$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | .1 | | .5 | | -.8 |
| 0 | 0 | 0 | 0 | 0 | .1 | 0 | .5 | 0 | -.8 |
| 0 | 1 | 1 | 0 | 0 | .1 | .2 | .7 | .2 | -.6 |
| 1 | 0 | 1 | 0 | .2 | .3 | 0 | .7 | .2 | -.4 |
| 1 | 1 | 1 | 1 | 0 | .3 | 0 | .7 | 0 | -.4 |
| 0 | 0 | 0 | 0 | 0 | .3 | 0 | .7 | 0 | -.4 |
| 0 | 1 | 1 | 1 | 0 | .3 | 0 | .7 | 0 | -.4 |
| 1 | 0 | 1 | 0 | .2 | .5 | 0 | .7 | .2 | -.2 |
| 1 | 1 | 1 | 1 | 0 | .5 | 0 | .7 | 0 | -.2 |
| 0 | 0 | 0 | 0 | 0 | .5 | 0 | .7 | 0 | -.2 |
| 0 | 1 | 1 | 1 | 0 | .5 | 0 | .7 | 0 | -.2 |
| 1 | 0 | 1 | 1 | 0 | .5 | 0 | .7 | 0 | -.2 |

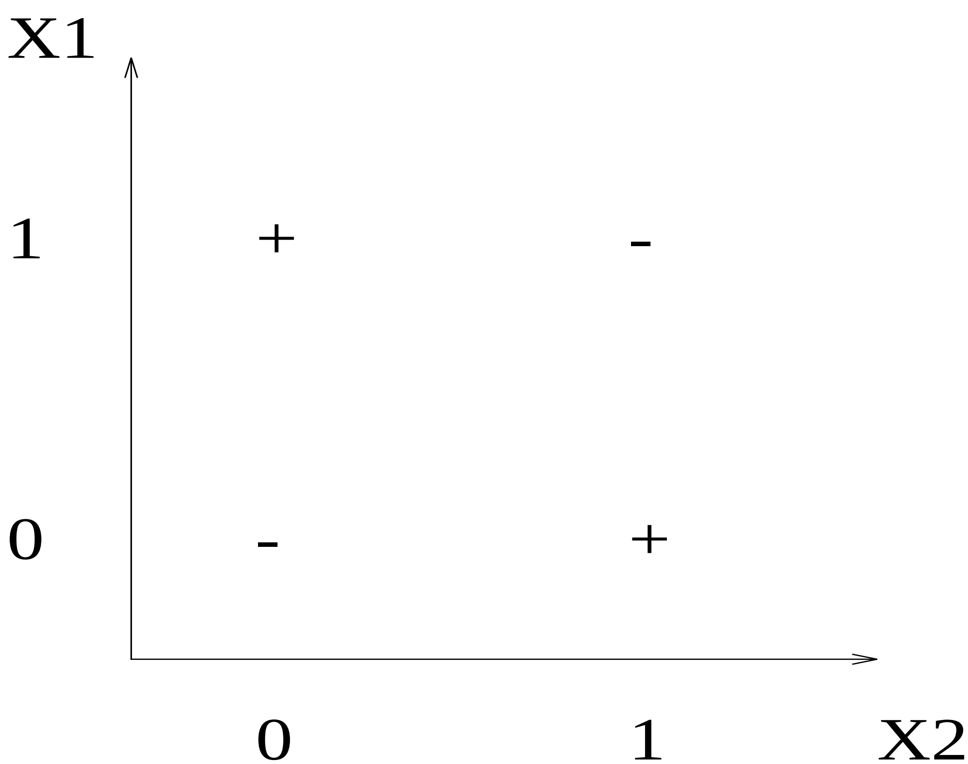
"If a set of I/O pairs is learnable, then the Perceptron Learning Rule will find the necessary weights."

Minsky and Papert, 1988.

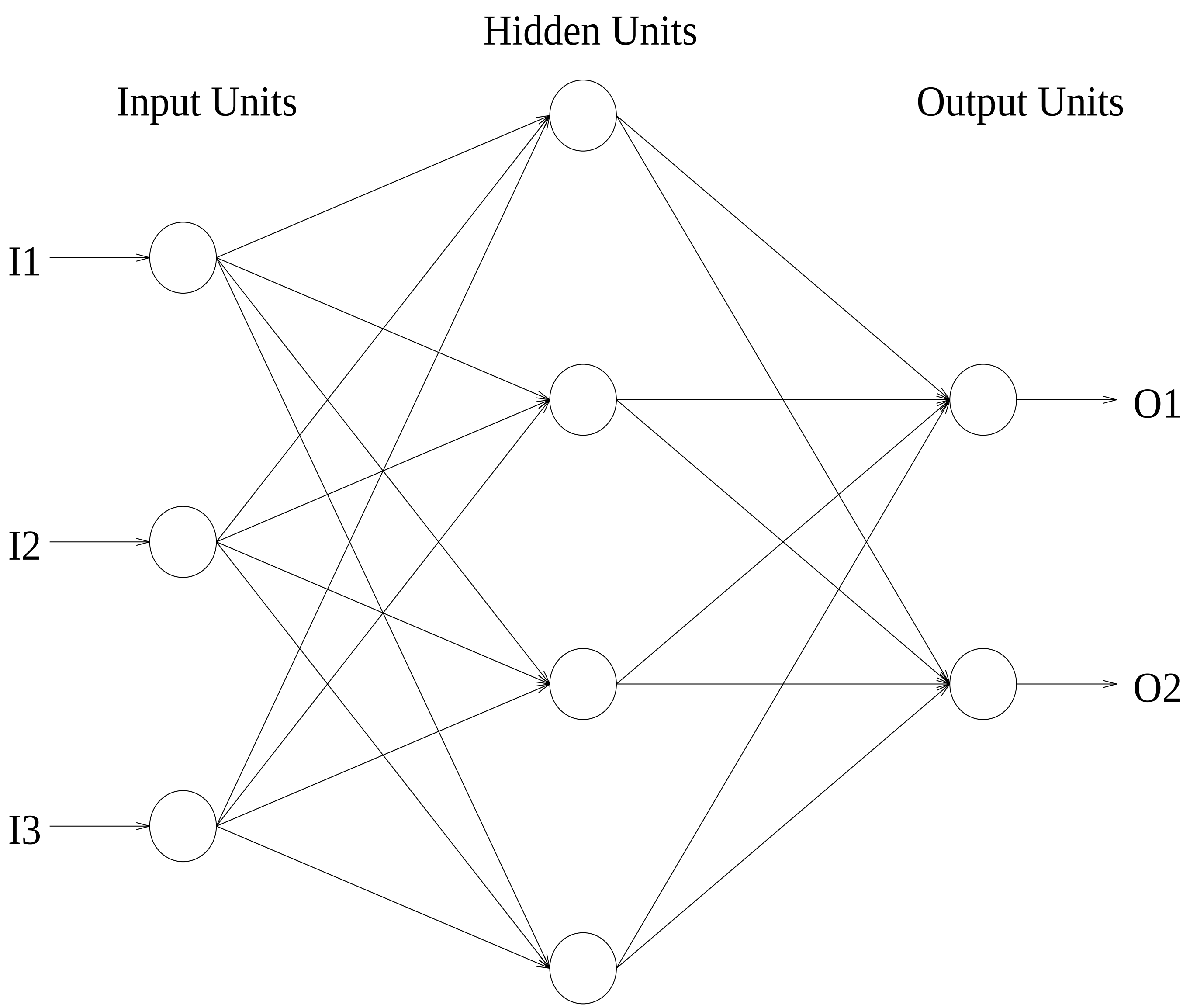
The output function $W_1X_1 + \ldots + W_nX_n > T$ defines a hyperplane that splits the input space into two half spaces.



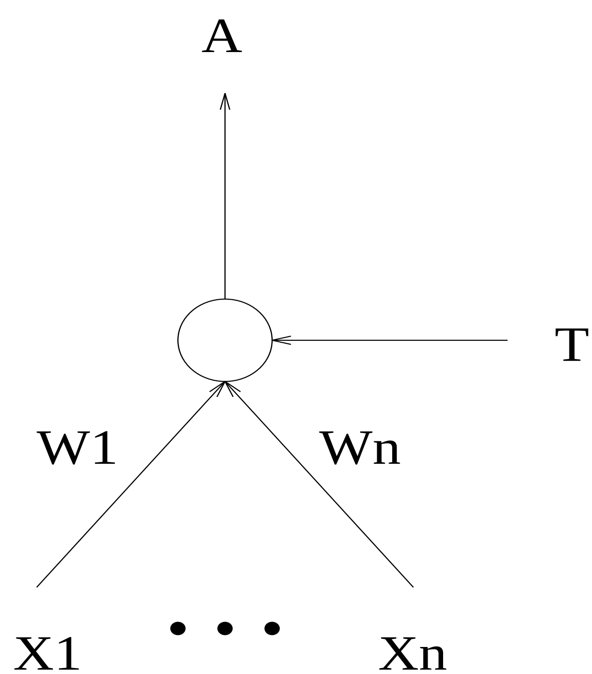
Can a single line separate the two classes?



# Solution: Hidden Units

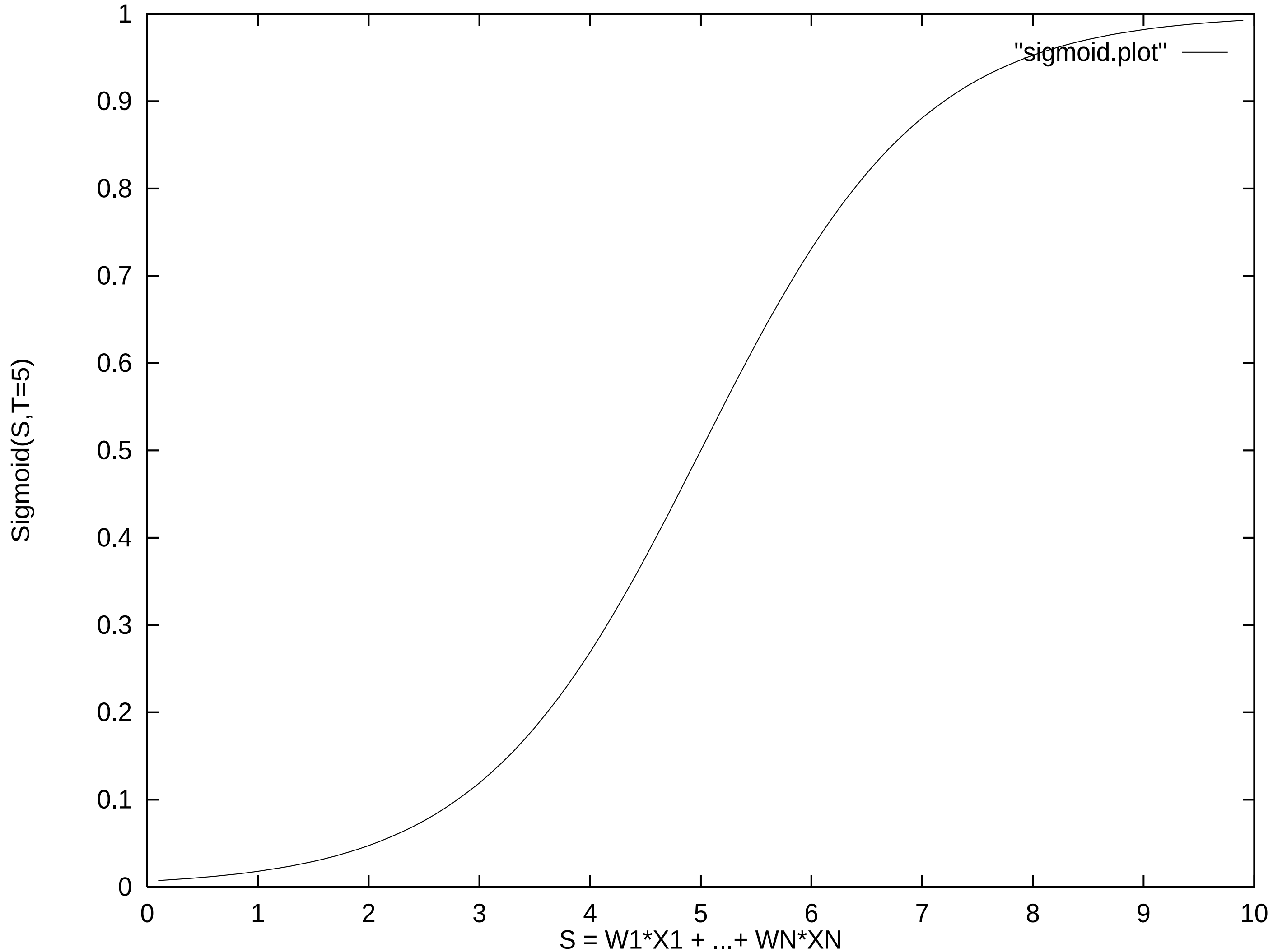


- Network contains hidden units.
- Each unit obeys "Sigmoidal Activation Function".

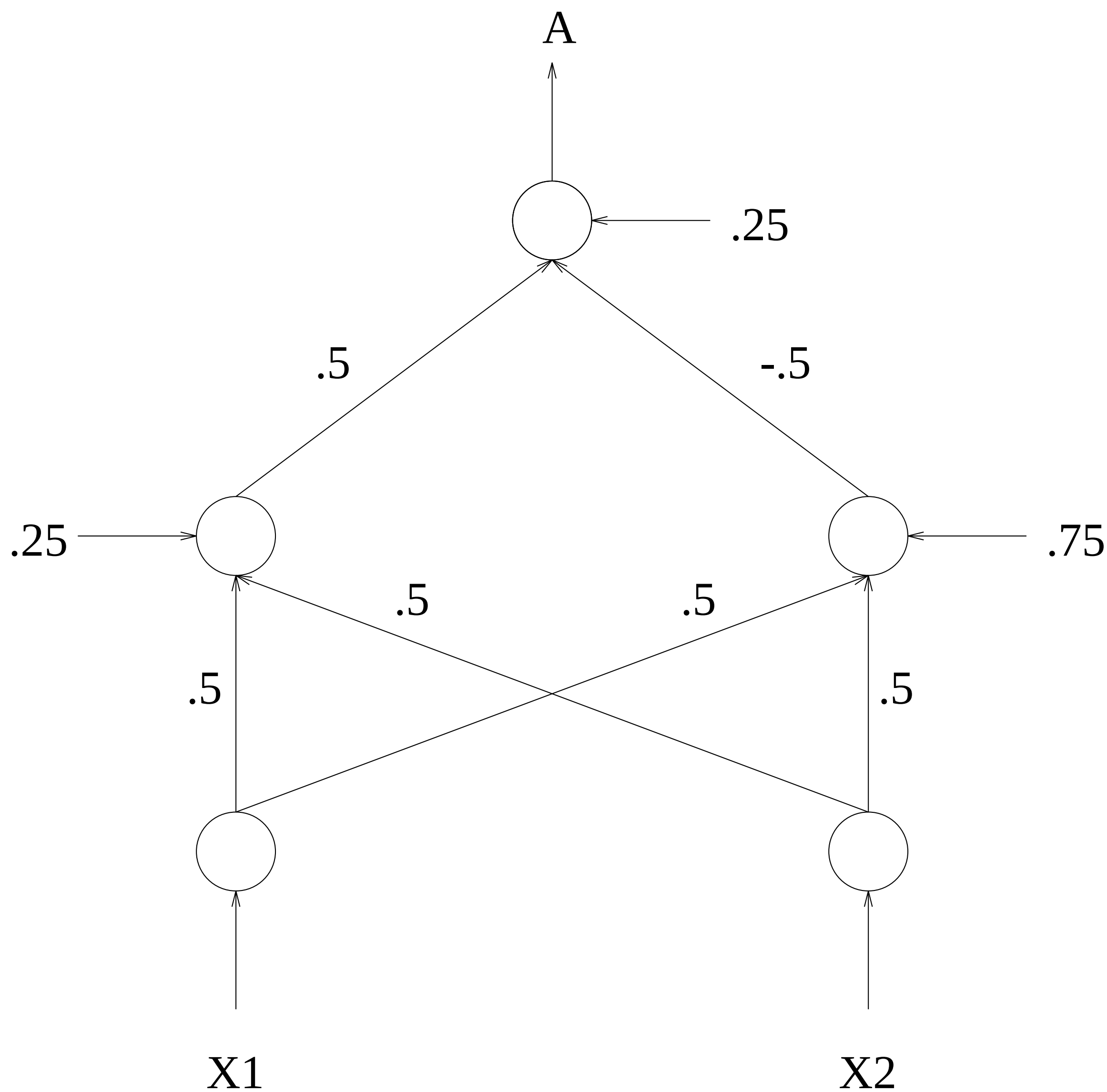



$$A = \frac{1}{1+e^{-(W_1X_1+\ldots+W_nX_n-T)}}$$

# Behavior of Sigmoid Function



# Implementation of the $XOR$ Function



# Backpropagation Learning Rule for Weights of Edges from Hidden Units to an Output Unit $j$

$$\Delta W_{ij} = \eta A_j(1 - A_j)(D_j - A_j)H_i$$

$H_i$ is a node's input (a hidden node's output).

$W_{ij}$ is the corresponding weight.

$\Delta W_{ij}$ is the change in weight.

$D_j$ is the desired output.

$A_j$ is the actual observed output.

$\eta$ is the learning rate.

# Backpropagation Learning Rule for Weights of Edges from Input Units to a Hidden Unit $i$

$$\Delta W_{ki} = \eta H_i(1 - H_i)E_i X_k$$

$X_k$ is a node's input.

$W_{ki}$ is the corresponding weight.

$\Delta W_{ki}$ is the change in weight.

$E_i = \Sigma_{j=1}^{n} W_{ij} A_j(1 - A_j)(D_j - A_j)$ is the propagated error from the $n$ output units.

$H_i$ is the actual observed output.

$\eta$ is the learning rate.

# Motivation for the Backpropagation Learning Rules

- The weights $\bar{W} = (W_1, \ldots, W_n)$ define a point in an $n$-dimensional Euclidean space.
- Each point $\bar{W}$ in the space defines a network.
- Each point $\bar{W}$ has an associated error rate:

$$E = \frac{1}{2}\Sigma_i(D_i - A_i)^2$$

- We compute the gradient $\nabla E$ with respect to $\bar{W}$.
- Each learning iteration changes $\bar{W}$ to $\bar{W} - \eta\nabla E$.
- A “Gradient Descent” algorithm.

- NETtalk converts character strings to phonemes.
- Neurogammon won the 1989 Computer Olympiad.
- ALVINN steers a vehicle along a single lane highway.

**Advantages:**

- Generalization capability.
- Low sensitivity to noise.

**Disadvantages:**

- Relative expressiveness.
- Computational efficiency.
- Transparency (black box).
- Hard to use prior knowledge.